Socio Econo Ensamble (SEE Version 1.0.0)

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Abstract: Model to …

Basics

N persons

ε_i wealth of a person i

E is the total wealth of the system

$$\longrightarrow \quad E = \sum \varepsilon_i$$

The question is how is the wealth distributed among the N persons or witch is the probability that a person has a wealth between ε and ε + dε

$$p(\varepsilon)d\varepsilon = ?$$

Hypothesis

The allocation of resources to the N different people:

$$(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \ldots \varepsilon_N)$$

with

$$E = \sum_i \varepsilon_i$$

is a possible state of the System.

My hypothesis is that each possible state is equal alike.

The set of different possible states is called in physics an “Ensemble”

If the total wealth is kept constant we speak of a “Canonical Ensemble”.

Counting states

Number of states

Total wealth

$$\Omega_N(E)$$

People

Simple case:

$$\Omega_1(E) = 1$$

Wealth split in ΔE wealth steps delivering M possible states:

$$M = \frac{E}{\Delta E}$$

Counting states

Number of states for N = 2 distinguible people

$$\Omega_2(E) = M$$

Number of states for N = 3

$$\Omega_3(E) = \sum_{m=0}^{M} \Omega_2(E - m\Delta E) = \frac{M\,(M+1)}{2}$$

Number of states for N = 4

$$\Omega_4(E) = \sum_{m=0}^{M} \Omega_3(E - m\Delta E) = \frac{(M-1)M(M+1)}{1 \cdot 2 \cdot 3}$$

Counting states

General equation for N people

$$\Omega_N(E) = \frac{1}{(N-1)!}\frac{(M+1)!}{(M-N+2)!}$$

In case M >> N and N >> 1:

$$\frac{(M+1)!}{(M-N+2)!} = \frac{(M+1)!}{(M+1-(N-1))!} \approx M^{N-1}$$

reduce to

$$\Omega_N(E) = \frac{1}{(N-1)!}\left(\frac{E}{\Delta E}\right)^{N-1}$$

Probability

Probability of a person to have wealth between ε and ε + dε:

$$p(\varepsilon)d\varepsilon = \frac{\Omega_{N-1}(E - \varepsilon)}{\Omega_N(E)}$$

In case N >> 1 with

$$\bar{\varepsilon} = \frac{E}{N} \qquad \text{or} \qquad \beta = \frac{N}{E}$$

the probability is

$$p(\varepsilon)d\varepsilon = \frac{1}{\bar{\varepsilon}}\, e^{-\varepsilon/\bar{\varepsilon}}\,\Delta\varepsilon \;=\; \beta\, e^{-\beta\varepsilon}\, d\varepsilon$$

People in a state

The allocation of resources in n states:

$$(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \ldots \varepsilon_n)$$

with N people distributed in each state:

$$(n_1, n_2, n_3, n_4, \ldots n_n)$$

Satisfying

$$E = \sum_i n_i \varepsilon_i \quad \text{and} \quad N = \sum_i n_i$$

Number of people in a state + Lagrange parameter α

$$\overline{n_s} = \frac{\sum_{n_1,n_2,..} n_s \, e^{-\beta(n_1\varepsilon_1 + n_2\varepsilon_2 + n_3\varepsilon_3 + ..) + \alpha}}{\sum_{n_1,n_2,..} e^{-\beta(n_1\varepsilon_1 + n_2\varepsilon_2 + n_3\varepsilon_3 + ..) + \alpha}} = \frac{\sum_{n_s} n_s \, e^{-\beta n_s \varepsilon_s + \alpha}}{\sum_{n_s} e^{-\beta n_s \varepsilon_s + \alpha}}$$

Boson Statistics

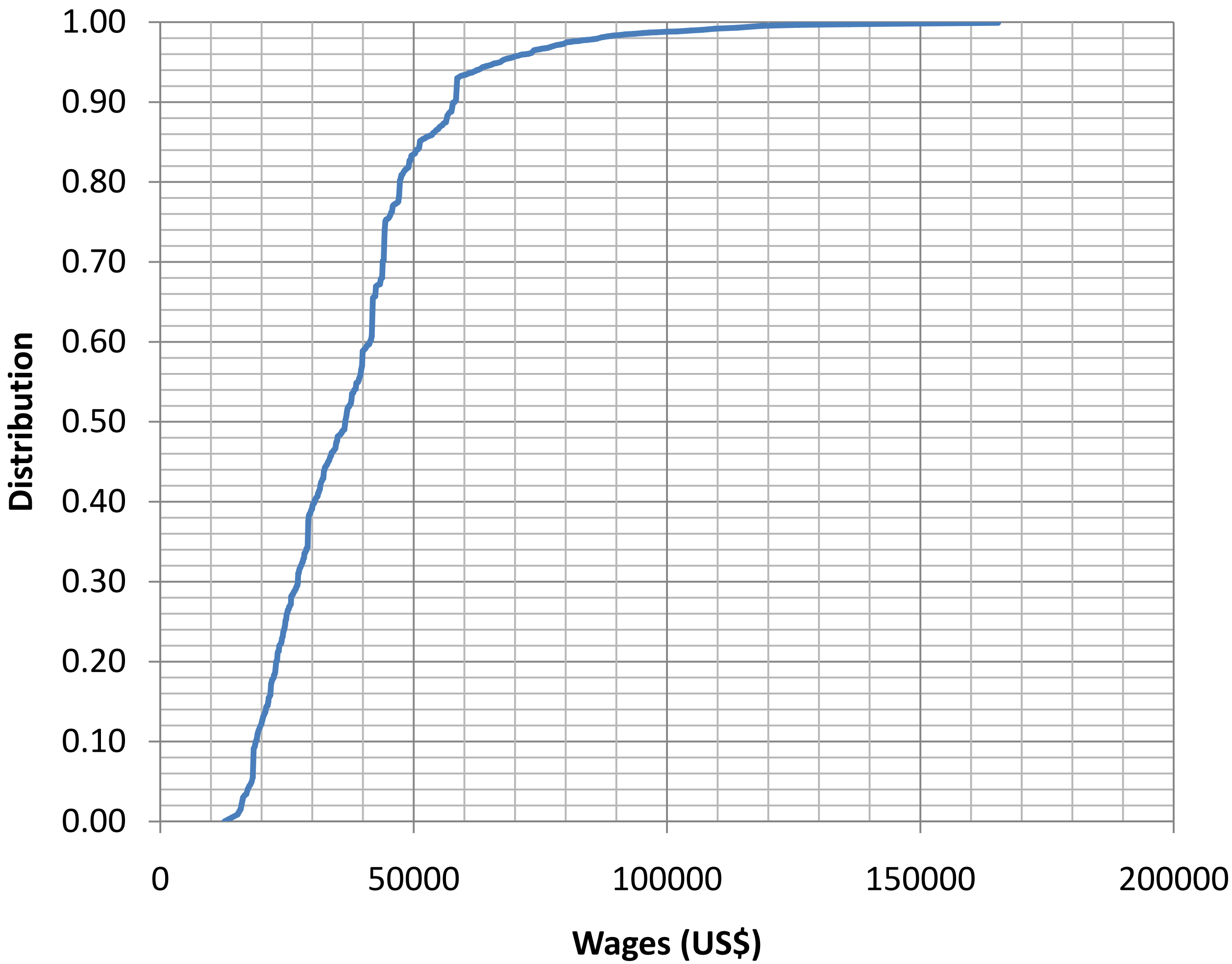
Earning like a Boson $n_s = (0,1,2,3,4,\ldots)$

$$n_s = \frac{\sum_{n_s} n_s \, e^{-\beta n_s \varepsilon_s + \alpha}}{\sum_{n_s} e^{-\beta n_s \varepsilon_s + \alpha}} = \frac{1}{e^{-\beta \varepsilon_s + \alpha} - 1}$$

α is in physics the “chemical potential” and has to be chosen to fulfill:

$$N = \sum_i n_i$$

Wages distribution (US)



Data: OES statistics for the US market in 2005

Fermion Statistics

Purchasing like a Fermion:

Product with a price ε_s
I'm buying the product $n_s = 1$
I'm not buying the product $n_s = 0$

Number of people purchasing the product "if you have no other choice":

$$n_s = \frac{\sum_{n_s} n_s \, e^{-\beta n_s \varepsilon_s + \alpha}}{\sum_{n_s} e^{-\beta n_s \varepsilon_s + \alpha}} = \frac{1}{e^{-\beta \varepsilon_s + \alpha} + 1}$$

Fermion Statistics

α to be chosen to fulfill:

$$N = \sum_i n_i$$

$$n_s = \frac{1}{e^{-\beta\varepsilon_s + \alpha} + 1}$$

This equation is known in marketing as the "Logistic Distribution". Its an empirical equation that describes the probability that a product is sold.

At $-\beta\varepsilon_s + \alpha = 0$
or $\varepsilon_s = \alpha/\beta$

$$n_s = 0.5$$

At a value equal α/β half of the customers will purchase the product. I will call this factor the "product value". It's the price the customer "thinks" will be ok.

Fermion Statistics

But in sociology the decision is not only price; there are additional criteria's (c_1,c_2,c_3, …) that people take to account in there decision. This mean that we must extend the ensemble to the decision criteria of the people:

$$(\varepsilon, \vec{c})$$

And I can introduce a new quantity called wellbeing ω_s defined by

$$\omega_s = \varepsilon_s + \vec{\eta} \cdot \vec{c}$$

In marketing studies η is calculated from the survey and represents what relevance the interviewed people assigned to the particular criteria c.

Fermion Statistics

With the extension of the ensemble to the decision criteria I arrive at the classical Logistic Distribution:

$$n_s = \frac{1}{e^{-\beta\omega_s + \alpha} + 1}$$

There model not only delivers the known distribution, it also adds to new insides. First, the criteria term

$$\vec{\eta} \cdot \vec{c}$$

becomes a “truth price” in a natural way, and second, the introduction of α allows to model the comparison between different alternatives n_{s1} , n_{s2} , n_{s3} the Logistic Distribution are not defining how to estimate.

From here

Erik/Raquel:

- Partition Function
- Hamiltonian
- Terrorism

Theo:

- Gini
- BE Condensation - unemployment